The research project has been implemented within the framework of the Action "Supporting Postdoctoral Researchers» of the Operational Program "Education and Lifelong Learning" (Action's Beneficiary: General Secretariat for Research and Technology), and co-financed by the European Social Fund (ESF) and the Greek State (SH4_3510 "EDARCPADSS").

SHORT REPORT

EPISTEMOLOGICAL AND DIDACTICAL ASPECTS RELATED TO THE CONCEPT OF PERIODICITY ACROSS DIFFERENT SCHOOL SUBJECTS

Chrissavgi Triantafillou

Post doctoral researcher, ASPETE, Branch of Patras, Pedagogical Department

Vasiliki Spiliotopoulou

Scientific advisor, ASPETE, Branch of Patras, Pedagogical Department

INTRODUCTION

Our aim in this study is to identify epistemological and didactical aspects related to the concept of periodicity. Periodicity is an essential scientific concept for the following reasons: it plays a central role in the school curriculum in various school subjects and it is fundamental in many undergraduate courses in mathematics, science and engineering. We can meet this notion throughout the secondary school curriculum and in undergraduate studies in relevant fields.

Our methodological plans covered three components of the school pedagogical practices and they in turn, were our three research phases. In the first phase of our study, we analysed the content presentation and the proposed exercises sections in Greek science and mathematics textbooks. In the second phase, our main interest was on how undergraduate students in scientific directions departments perceive periodic motions and their graphical representations. Our focus in the third phase was on secondary teachers' practices when they teach topics relative to periodicity in their classes.

From textbook analysis, the crucial role of argumentation in conceptualising periodicity emerged. Hence, we investigated how secondary teachers enact on the textbook argumentation and we tried to identify how these two factors, textbooks and teachers' practices, could contribute to or limit students' conceptualisation of the notion.

BACKGROUND & RESEARCH QUESTIONS

Although a scientific or mathematical notion is independent of any given text, its expression and understanding requires the use of discipline-specific textual conventions and practices to make sense of print and visual components of the text (Yore, Pimm, & Tuan, 2007) and to identify and link rhetorical elements that form the argument of the target notion (Lin & Yang, 2007). Reasoning, the human capacity to make sense of the world, has long been the goal of science and mathematics. Despite the obvious differences in the two subjects' themes, Lakatos (1976) pointed out the strong parallels between mathematical and scientific reasoning discourse. Furthermore, understanding the target notion in a text in mathematics and science involves images of instances of the notion. These representations in a school text are expressed either visually (e.g. pictures, diagrams or maps) or symbolically (e.g. equations or formulae). We consider that representations and arguments related to the notion in school texts are both resources which act as bearers of distributed intelligence and that they carry, in a compressed way, socio-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1995; Pea, 1993). These ubiquitous mediating structures both organise and constrain teachers' teaching practices and provide to students a specific, conceptually structured space to think (Rezat & Strässer, 2012).

In first phase of our study, we adopted the position that textbooks aim to introduce their readers to the conceptual aspects of scientific and mathematical knowledge and persuade them of their value. By restricting our attention to thematic units related to the notion of periodicity in Greek mathematics and science textbooks, we addressed the following research questions:

- How is argumentation structured and developed when employed in the texts on periodicity?
- How is argumentation unfolded and co-deployed with conceptual aspects of periodicity while reading a thematic unit?
- In what respect are argumentation practices similar or different in school mathematics and science textbooks?
- What are the conceptions of periodicity that may be stimulated by the solutions to exercises and problems in the given sample?

In the second phase of our study, we adopted the perspective that periodicity, as an abstract notion, is realised through specific situations where it takes its meaning (Radford, 2013). Under this theoretical perspective we designed three different research activities in which different situations of the notion (simple harmonic oscillation; periodic and non-periodic motions; springs in car-suspension system) are involved which are addressed to undergraduate students. Our main research questions in this part of our study were:

- How do students interpret and connect textual and visual representations of periodic motions taken from their school textbooks? What type of difficulties do they meet when they have to make connections between the visual representations of different aspects of a periodic phenomenon?
- How do students interpret graphs of periodic motions and do they distinguish them from graphs of repeated but non-periodic motions?
- What are the ways in which students - prospective mechanical engineering teachers - transform a school text on periodicity into a teaching explanatory

unit? What levels of awareness and structural attention are revealed through this transformation in terms of identifying the functional relation between the scientific and the applied context?

In the third phase of our study, we sought to discover how educators in the various disciplines institutionalise their students' knowledge on aspects of periodicity and how they use texts' inherent logic when teaching aspects of periodicity. In order to investigate this general issue, we designed and conducted two research activities. Our main research questions were as follows:

- Which images are fundamental in teachers' teaching practices? How do they argue against students' misunderstanding of the periodic behaviour on non-periodical images?
- What is the role of everyday examples in their teaching practices? Do they follow or do they modify the knowledge organisation in texts in specific thematic units related to the notion of periodicity?
- What are their suggestions on how they could contribute to their students' development of a unified way of understanding periodicity?

Finally, through our research, we have tried to highlight aspects of the transfer issue from one school subject to the other, an issue that remains open and which challenges the didactical practices in formal education.

METHODOLOGY

Our research data

Our project took place from February 2013 to February 2015. Initially we analyzed 110 thematic units, 214 Visual Representations (VRs) and 162 proposed exercises taken from 11 Greek textbooks (mathematics, physics, astronomy & applied technologies). Our discussion in the final report will take place only about the subjects of mathematics and physics (72 thematic units, 184 VRs & 143 proposed exercises), since our initial analysis indicated that these are the main school subjects that introduce students to the notion of periodicity in Greek lower secondary and upper secondary general and vocational schools. Besides, 288 undergraduate students (230 from four different technological institutions and 58 from two different university institutions) participated in the three research activities. These activities were based on open-ended questionnaires. Finally, 50 teachers who teach mathematics, physics and engineering courses in general and vocational schools participated in two research activities (open-ended questionnaires & interviews).

Data analysis

We used a grounded theory research approach (Corbin & Strauss, 2008) partly in response to an increasing awareness of the limitations of applying a priori deductive theory to human transactions embedded in a social or an educational context, and partly in response to the lack of an existing scheme of categories broad enough to allow us to study how periodicity is presented and argued across mathematics and science textbooks.

Our analysis of school texts was done by creating two levels of content analysis. Initially, the unit of analysis is a thematic unit that refers to an aspect of the notion and maintains all the elements that are related to its conceptual understanding. In the second level, the unit of analysis are parts of the text in the thematic unit that could be considered as logical elements that develop a form of reasoning. These parts may correspond to one or more sentences and the accompanying visual representations and are characterised as *modes of reasoning* (MsoR).

In the first part of our study, inductive content analysis was applied on all thematic units and a coding system of categories and subcategories of MsoR was produced (Fig. 1). This framework provided us a set of filters through which we could systematically analyze data on selected tasks from physics and mathematics textbooks in terms of how knowledge is presented and argumentation is unfolded not only through the texts of units, but also through textbooks' proposed exercises; students' responses in selected tasks on periodicity and teachers' practices in terms of teaching periodicity and the use of textbooks' argumentation as well as their justifications on students' selected tasks.

CONCLUSIONS

The synthesis of our results from the three research phases provides us with evidence on the following issues:

In textbooks the co-deployment of argumentation and conceptualisation is inevitable, while understanding through reading is viewed as occurring through the dialectical relationship between these two channels of thought.

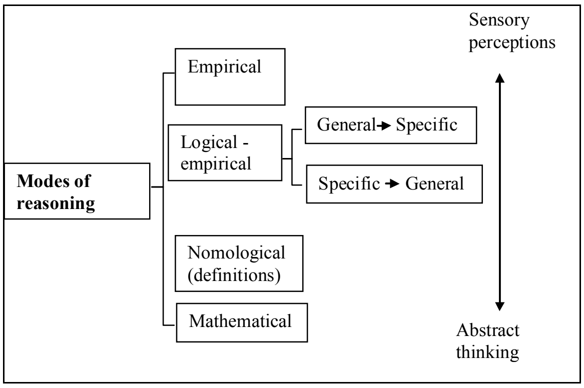

Fig. 1: The systemic network of the spectrum of Modes of reasoning (Bliss, Nonk & Ogborn, 1983) of knowledge development in Greek mathematics and physics texts.

Argumentation in thematic units is made through a series of syllogisms or MsoR (Triantafillou, Spiliotopoulou & Potari 2015); the kind of visual representations (Vrs); and their co-deployment (Triantafillou, Spiliotopoulou & Potari, 2013a & 2013b· Spiliotopoulou & Triantafillou, 2014· Triantafillou & Spiliotopoulou, 2014). These syllogisms belong in a spectrum from sensory perceptions (empirical & logical-empirical MsoR to syllogisms of abstract thinking (nomological & mathematical).

The empirical MsoR attempt either to direct the readers' attention to recall experiences with periodic motions in real-life phenomena, or to describe these in a systematic way so that periodicity may be considered as being closer to the readers' perceptions. The logical-empirical MsoR link empirical evidence with general outcomes; therefore, studying the dynamic features of certain instances of periodic phenomena could lead to understanding their general characteristics or vice versa. The nomological MsoR indicate the epistemological and ontological aspects of periodicity aimed to be learned in each subject. Finally, the mathematical MsoR support abstract ways of thinking by providing supports to scientific claims or mathematical claims where representations of periodicity are mostly symbolic in nature.

In terms of Visual Representations' genre, the photos and naturalistic drawings of periodic motions help readers to visualise everyday periodic phenomena while the schematic representations and the graphs mostly support abstract ways of thinking about the notion. Mathematics textbooks avoid using photographs (that represent particular instances) and the use of images that convey a generality is preferred. We argue that this practice is purposeful, since maths authors seem to believe that these images do not support a 'proper mathematical' argument (Herbel-Eisenmann & Wagner, 2007). Although recent research has stressed the decisive and prominent role of bodily actions and gestures in students' elaboration of elementary, as well as abstract mathematical knowledge (Nunez, 2007), this is neglected by the authors of Greek maths textbooks. In contrast, physics textbooks (mostly for lower levels) try to bring the notion closer to readers' empirical experiences (Triantafillou & Spiliotopoulou, 2013, published in Greek). This approach, however, is not used in the 'textbook proposed exercises' sections, since most of the exercises in physics and almost all the exercises in mathematics texts on periodicity are context-free. The above approaches could affect students' understanding of the target notion. This is because these approaches tend to narrow the way students engage with modelling activities that make use of real life periodic phenomena.

We note that the absence of particular MsoR could result in reasoning gaps which may influence understanding of the argumentation developed in the text (e.g., we noticed that in specific thematic units on simple harmonic oscillations, the sinusoidal curve comes arbitrarily, see Triantafillou, Spilitopoulou & Potari, 2015). We argue that if readers do not pass through certain MsoR resulting in the sinusoidal curve, then important conceptual or logical elements may be missing. In this case, teachers usually enact on this practice by adding the missing modes MsoR. We argue that teachers' modifications of the inherent logic of a thematic unit could be made for several reasons (e.g., when valuing that a MoR is didactically inappropriate or in order to stimulate their students' attention). Additionally, teachers mentioned that they enrich their teaching with everyday images and examples of periodic phenomena beyond the ones provided in the text. We argue that teachers' appropriate modifications might sustain students' visualisation of periodic motions (e.g., by supporting logical-empirical MsoR with the use of digital tools) or add to their understanding of text argumentation by covering reasoning gaps in texts (e.g., physics teachers involve the trigonometric circle in order to reason about the sinusoidal curve). In some cases, though, the teacher unconsciously might limit the inherent logic of the concept presentation of the text, either by omitting specific MsoR that are important in the argumentation developed in a thematic unit, or by not placing the everyday examples of periodic phenomena as integral parts of the argumentation developed in the thematic unit. On the one hand, this fact highlights the importance of teachers' appreciation of all syllogisms - from sensory perceptions to abstract thinking and reasoning - as being important rational actions in concept formation. On the other hand, teachers see the need to make transparent to their students the distinction between the different roles of the above range of syllogisms in the development of a sound argument.

Visual representations as images of periodicity play a major role in the inherent logic of the concept presentation in texts either by supporting a method of reasoning, or by being a MoR outcome. The main images of periodicity are the sinusoidal curve and the trigonometric circle. These two images are linked, since the sinusoidal curve comes as a result of a series of MsoR that are based mostly on the trigonometric circle (a reasoning practice common in mathematics and physics, see Triantafillou, Spiliotopoulou & Potari, 2013a). The sinusoidal curve is acknowledged by all teachers as their main teaching tool, while 20% of images in all school texts analysed are sinusoidal graphs (Spiliotopoulou & Triantafillou, 2014). For all these reasons, we consider the sinusoidal curve as the prototypical image of periodicity across subjects. This comes in conflict with undergraduate students' misconceptions on the sinusoidal curve. Particularly, students consider that the sinusoidal curve (f(x) = sinx) shares the periodical behaviour with the function $f(x) = e^{-bx} \sin(x)$. Possible explanations of this misunderstanding could be either because in physics texts, the distinction of the two functions in terms of their periodical behaviour is not so clear, or because of the absence of examples of repeated but non-periodic motions in texts (Triantafillou, Spiliotopoulou, Sideris & Kexrakos,, 2012, published in Greek). These types of examples help students to make comparisons and facilitate understanding of main characteristics of the notions by creating rich conceptual representations (Bills, Dreyfus, Mason, Tsamir, Watson & Zaslavsky, 2006).

In such cases the periodical curve is considered as a 'mathematical tool' that is used in a new situation, or we might say that it transfers from a mathematical to a non-mathematical subject. We argue that this perception underestimates the role of this 'mathematical tool'. Students do not only transfer a 'tool' (the sinusoidal curve), but rather the whole range of syllogisms that produce this 'tool'. Of course, the issue that this tool is used in different contexts makes the transfer more difficult, but in our view this is not the only explanation for the transfer of knowledge is a challenging activity for every student. Students do not transfer from one epistemological field to the other merely 'tools', but complex structures of argumentation and reasoning.

Our study highlights *ontological differences* between physics and mathematics when ascending from observations to generalisations. Particularly, mathematical evidence-based reasoning seems to be safer and more reliable than experimental evidence-based reasoning. The role of evidence - circumstantial or supportive - seems to be a main issue in scientific reasoning and the tentative nature of science (Ohertman & Lawson, 2008). Above and beyond their differences, we argue that mathematics and physics share *common pedagogical practices* (e.g., engage students in argumentation and reasoning and use common images on periodicity, such as the sinusoidal function). Further, maths and physics teachers share *common reasoning behaviours* when they argue on the validity of students' justifications. Despite the above commonalities between the subjects of mathematics and physics, students seem to face many difficulties when there is a need to integrate knowledge from the two subjects and overcome conflicts among them (e.g., misconceptions about the function $f(x) = e^{-bx}\sin(x)$). This creates a necessity for mathematics and physics teachers to help their students to develop a unified view on periodicity where aspects of the notion from the different subjects coexist harmonically. Teachers made some suggestions on the above necessity (e.g., provide many examples of everyday life periodic phenomena; make links of the concrete situations with the mathematical objects by using animations of period motions). The above teachers' suggestions are important, however, we notice that the practice of co-operation among colleagues of mathematics and physics seems to be almost neglected by teachers. Connecting mathematics and physics instruction is considered as a central issue in the contemporary research literature (Frykhlom & Glasson, 2005) since it can strengthen students' understanding of common and neighbouring notions. Furthermore, we argue on the complexity of the engineering context, in which the above connections are not an option for teachers and students - but rather obligatory practices.

Argumentation and reasoning seem to be unfamiliar practices for undergraduate students. In the case of participating in this type of practice, mostly logical-empirical types of justifications were identified in the students' responses, and only rarely did they use formal definitions as warrants in their responses (e.g., even mathematics undergraduate students avoid using the formal mathematical definition for periodic functions). One reason for students' reluctance to argue on their claims could be either because argumentative-pedagogy is not a common practice in the Greek educational system or because certain conceptual elements are not consciously comprehensible by them. We provided evidence that students' explanations became more sophisticated when moving from partial to sufficient explanations, while this differentiation depends on students' abilities to express connections between different fields of knowledge in an argumentative way (Triantafillou & Spiliotopoulou, in press). In this case, context-dependency is an action that fosters students in developing a more articulated, and thereby more elaborated understanding of the notion.

DIDACTICAL IMPLICATIONS OF OUR STUDY

Findings can inform school textbooks' authors as the study results highlight the importance of the argumentation in students' conceptualisation of periodical phenomena. Teachers' awareness of textbooks' reasoning practices can play an important role in teaching interventions. Especially, mathematics and physics teachers need to discern the ontological and epistemological differences of science and mathematics textbooks in terms of periodicity in order to be able to fill in reasoning and conceptual gaps. To our view, practices that could help students in this direction could be: students' participation in a wide range of reasoning practices (from sensory perceptions to abstract thinking and reasoning) on periodicity where they could overcome conflicts and conjectures; students' participation in contextual activities with real life periodic phenomena (e.g., modelling with ferry wheel); and teachers' familiarisation with cooperation practices with their colleagues in neighbouring subjects. This co-operation could involve sharing teaching experiences and familiarising each other perceptions about the notion.

In general, in understanding periodicity the epistemological context is influenced by the pedagogical context (textbooks; teachers' practices; and conventions in each epistemological field). The co-existence of all the above perceptions does not favour students' development of a functional and articulated knowledge on periodicity. Some of our suggestions in this direction are: Developing innovative classroom material linked with contemporary learning and teaching theories that could initiate students' integration of the two cultures of inquiry (mathematical and scientific); Developing inquiry-based learning activities in our classrooms (Maaß & Artigue, 2013); developing interdisciplinary communities of inquiry (Jaworski, 2006). The above could contribute to students' understanding of a scientific notion that is an integral part of many scientific fields and help them to connect conceptions from different epistemological fields and apply their knowledge in a repertoire of reference practices.

REFERENCES

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. In J. Novotná, H. Moraová, M.

Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 126-154). Prague, Czech Republic: PME.

Bliss, J., Monk, M., & Ogborn, J. (1983). *Qualitative data analysis for educational research*. London, England: Croom Helm.

Corbin, J. M., & Strauss, A. L. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: SAGE.

Dolev, S., & Even, R. (2013). Justifications and explanations in Israeli 7th grade math textbooks. *International Journal of Science and Mathematics Education.* Advance online publication. doi:10.1007/s10763-013-9488-7.

Frykholm, J. & Glasson, G. (2005). Connecting Science and Mathematics Instruction: Pedagogical Content Knowledge for Teachers. *School Science and Mathematics*. *105*(3), 127-141.

Herbel-Eisenmann, B. & Wagner, D. (2007). How a textbook might position a mathematics learner. *For the Learning of Mathematics*, *27*(2), 8-14.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9(2), 187-211.

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, England: Cambridge University Press.

Lektorsky, V. A. (1995). Knowledge and cultural objects. In L. Kuçuradi & R. S. Cohen (Eds.), The concept of knowledge. The Anakara seminar (pp. 191–196). Dordrecht: Kluwer.

Lin, F. L., & Yang, K. L. (2007). The reading comprehension of geometric proofs: The contribution of knowledge and reasoning. *International Journal of Science and Mathematics Education, 5*(4), 729-754.

Maaß, K. & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis, *ZDM*, 45(6), pp 779-795.

Nunez, R. (2007). The cognitive science of mathematics: Why is it relevant for mathematics education? In R. A. Lesh, E. Hamilton & J. Kaput (Eds.), *Foundations for the future in mathematics education*. London: Lawrence Erlbaum Associates.

Oehrtman, M., & Lawson, A. E. (2008). Connecting science and mathematics: The nature of proof and disproof in science and mathematics. *International Journal of Science and Mathematics Education, 6,* 377- 403.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), Distributed cognitions (pp. 47–87). Cambridge: Cambridge University Press.

Radford, L. (2013). On the role of representations and artefacts in knowing and learning, Educational studies in mathematics, DOI 10.1007/s10649-013-9527-x.

Rezat, S., & Strässer , R. (2012). From the didactical triangle to the socio - didactical tetrahedron: Artifacts and fundamental constituents of the didactical situation. ZDM, 44, 641-65.

Spiliotopoulou, V. & Triantafillou, C. (2014). Visual Representations of the Concept of Periodicity across Subjects: Their Context and Genre. In C. P. Constantinou, N. Papadouris & A. Hadjigeorgiou (Eds.), *E-Book Proceedings of the ESERA 2013 Conference: Science Education Research For Evidence-based Teaching and Coherence in Learning*. Part 2, (pp. 334-344) Nicosia, Cyprus: European Science Education Research Association. ISBN: 978-9963-700-77-6.

Spiliotopoulou, V. & Triantafillou, C. (in press). Engineering students' teaching plans on periodicity: transforming school texts, *Proceedings of NDSTE 2014 conference*, Cambridge: Cambridge Scholars Publishing.

Triantafillou, C. & Spiliotopoulou, V. (2013). A method of analyzing argumentation in school texts: The case of periodicity in Physics texts (Μια μέθοδος ανάλυσης της επιχειρηματολογίας σχολικών κειμένων: Η περίπτωση της περιοδικότητας στη Φυσική). *Proceedings of the 8th Panhellenic Conference of Didactics in Science and New technologies in Education* (*Πρακτικά 8ου Πανελλήνιου Συνεδρίου Διδακτικής των Φυσικών Επιστημών και Νέων Τεχνολογιών στην Εκπαίδευση)*, (pp. 260-267),. Volos, April, 26-28. ISBN 987-618-80580-1-9 [in Greek].

Triantafillou, C. & Spiliotopoulou, V. (2014). Periodicity in mathematics and science texts: The co-deployment of visual representations and reasoning. In C. P. Constantinou, N. Papadouris & A. Hadjigeorgiou (Eds.), *E-Book Proceedings of the ESERA 2013 Conference: Science Education Research For Evidence-based Teaching and Coherence in Learning*. Part 2, (pp. 1200-1211). Nicosia, Cyprus: European Science Education Research Association. ISBN: 978-9963-700-77-6.

Triantafillou, C., Spiliotopoulou, V. & Potari, D. (2013a). Periodicity in textbooks. Reasoning and visual representations. In Lindmeier, A. M. & Heinze, A. (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 297-304. Kiel, Germany: PME.

Triantafillou, C., Spiliotopoulou, V. & Potari, D. (2013b). The nature of argumentation in school texts in different contexts. In Ubuz, B., Haser, Ç., & Mariotti, M. A. (Eds.). *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education (CERME)*, pp. 256-265. Antalya, Turkey: CERME.

Triantafillou, C., Spiliotopoulou, V. & Potari, D. (2014). How undergraduate students make sense out of graphs: the case of periodic motions. 2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) *Proceedings of the Joint Meeting of PME 38 and PME-NA 36,* Vol. 5, pp. 273-280. Vancouver, Canada.

Triantafillou, C., Spiliotopoulou, V., & Potari, D. (2015). The Nature of Argumentation in School Mathematics and Physics Texts: The Case of Periodicity. *International Journal in Mathematics and Science Education*, DOI 10.1007/s10763-014-9609-y.

Triantafillou, C., Spiliotopoulou, V., Sideris E. & Kexrakos D. (2012). The notion of periodicity in school textbooks and engineering students' perceptions (Η Έννοια της Περιοδικότητας στα Σχολικά Βιβλία και οι Αντιλήψεις των Σπουδαστών ΤΕΙ). *Proceedings in 29th Panhellenic Conference of Mathematics Education. (Πρακτικά 29ου Πανελληνίου Συνεδρίου Μαθηματικής Παιδείας)*, pp.841-855. EME: Branch of Kalamata [in Greek].

Yore, L. D., Pimm, D., & Tuan, H.-L. (2007). The literacy component of mathematical and scientific literacy. *International Journal of Science and Mathematics Education*, *5*(4), 559–589.